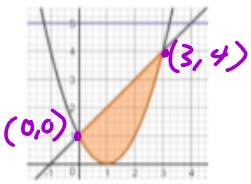
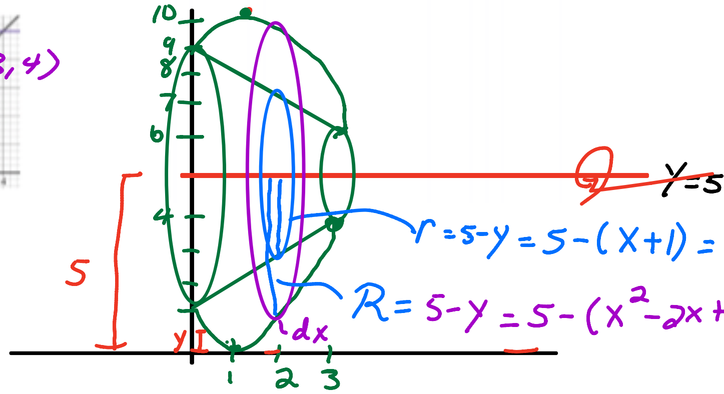
Period 3, March 5, 2025

6.

Find the volume of the solid generated by revolving the region bounded by the graphs of $y = x + 1$ and $y = x^2 - 2x + 1$ about the line $y = 5$.





$r = 5 - y = 5 - (x+1) = 4 - x$

$R = 5 - y = 5 - (x^2 - 2x + 1) = 5 - x^2 + 2x - 1$
$= 4 - x^2 + 2x$

$$\int_0^3 \pi (R^2 - r^2)\,dx = \int_0^3 \pi \left[(4 - x^2 + 2x)^2 - (4 - x)^2\right] dx$$

$$(4 - x^2 + 2x)(4 - x^2 + 2x) = 16 - \cancel{4x^2} + 8x - \cancel{4x^2} - \cancel{2x^3} + 8x - \cancel{2x^3} + \cancel{4x^2} + x^4$$

$$(16 - 4x^3 - 4x^2 + 16x + x^4)$$

$$(4 - x)^2 = 16 - 8x + x^2$$

$$\pi \int_0^3 \left[(16 - 4x^3 - 4x^2 + 16x + x^4) - (16 - 8x + x^2)\right] dx$$

$$\pi \int_0^3 \left[\cancel{16} - \cancel{4x^3} - \cancel{4x^2} + 16x + x^4 - \cancel{16} + 8x - \cancel{x^2}\right] dx$$

$$\pi \int_0^3 (x^4 - 4x^3 - 5x^2 + 24x)\,dx = \pi \left[\tfrac{1}{5}x^5 - x^4 - \tfrac{5}{3}x^3 + 12x^2\right]_0^3$$

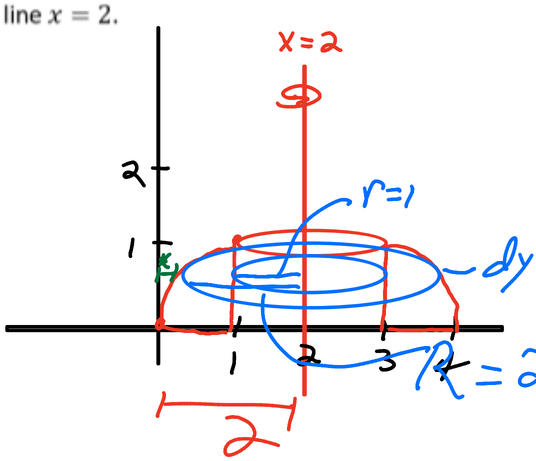
$$\left(\frac{243}{5} - 81 - \frac{5}{3}(27) + 12 \cdot 9 - \cancel{\left[\tfrac{1}{5}(0)^5 - 0^4 - \tfrac{5}{3}(0)^3 + 12(0)^2\right]}\right)\pi \quad (48\tfrac{3}{5} - 81 - 45 + 10$$

$\pi(156\frac{3}{5} - 126) = 36\frac{3}{5}\pi$

7.

Find the volume of the solid generated by revolving the region bounded by $y = \sqrt[3]{x}$, the x-axis, and the line $x = 1$ about the line $x = 2$.

$y^3 = x$



$R = 2-x$

$\int_0^1 \pi(R^2 - r^2)\,dy$

$\pi\int_0^1 [(2-x)^2 - 1^2]\,dy$

$\pi\int_0^1 [(2-y^3)^2 - 1]\,dy$

$\pi\int_0^1 [4 - 4y^3 + y^6 - 1]\,dy$

$\pi\int_0^1 [3 - 4y^3 + y^6]\,dy$

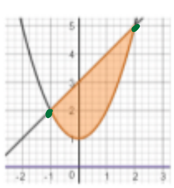
$\pi[3y - y^4 + \frac{1}{7}y^7]\Big|_0^1$

$\pi(3(1) - (1)^4 + \frac{1}{7}(1)^7] - \cancel{\pi[3\cdot 0 - 0^4 + \frac{1}{7}(0)^7]} = \pi(3 - 1 + \frac{1}{7})$

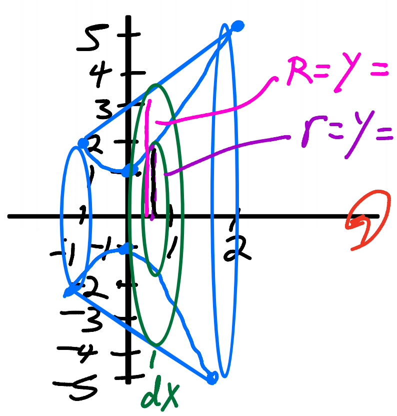
$2\frac{1}{7}\pi$

3.

Find the volume of the solid generated by revolving the region bounded by the graphs of $y = x^2 + 1$ and $y = x + 3$ about the x-axis.



$\int_{-1}^{2} \pi[(x+3)^2 - (x^2+1)^2]\,dx$



$$\pi\int_{-1}^{2}\left[(x^2+6x+9)-(x^4+2x^2+1)\right]dx$$

$$\pi\int_{-1}^{2}\left[x^2+6x+9-x^4-2x^2-1\right]dx$$

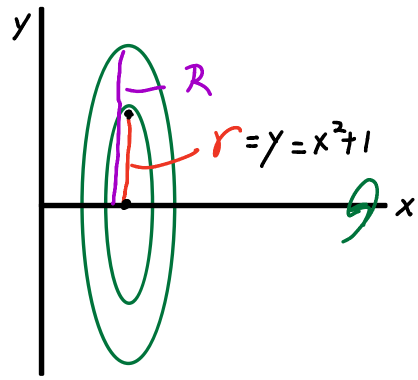
$$\pi\int_{-1}^{2}\left[-x^4-x^2+6x+8\right]dx$$

$$\pi\left[-\tfrac{1}{5}x^5-\tfrac{1}{3}x^3+3x^2+8x\right]\Big|_{-1}^{2}$$

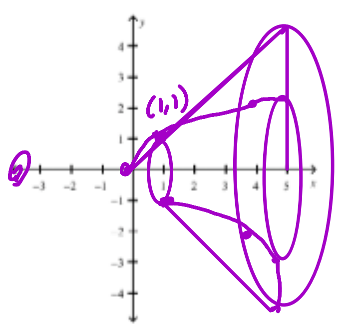
$$\pi\left[-\tfrac{1}{5}(2)^5-\tfrac{1}{3}(2)^3+3(2)^2+2\cdot 8\right]-\pi\left[-\tfrac{1}{5}(-1)^5-\tfrac{1}{3}(-1)^3+3(-1)^2+8(-1)\right]$$

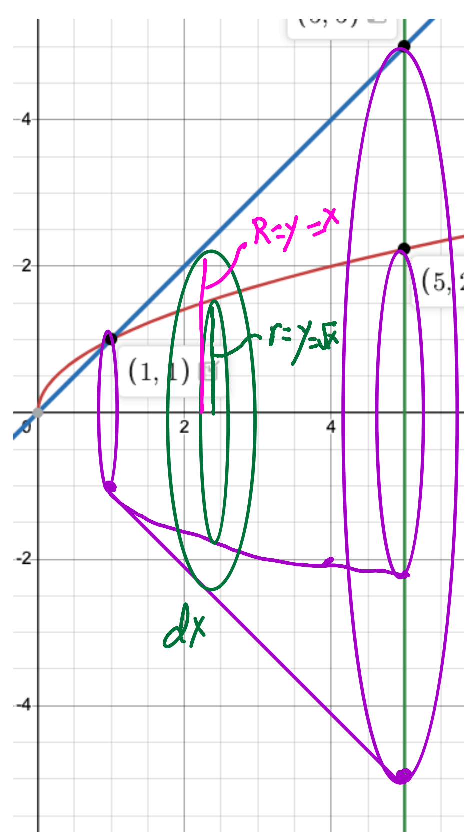
$$\pi\left[-\tfrac{32}{5}-\tfrac{8}{3}+12+16\right]-\pi\left[\tfrac{1}{5}+\tfrac{1}{3}+3-8\right]=\pi\left[-6\tfrac{2}{5}-2\tfrac{2}{3}+28-\tfrac{1}{5}-\tfrac{1}{3}-3+8\right]$$

keep going



2. Find the volume of the solid that results when the region enclosed by the curves $y=\sqrt{x}$, and $x=y$, **and $x=5$** is revolved about $y=0$. **[No Calc; solve and simplify this question by hand]**





$$\int_{1}^{5}\pi\left[R-r^2\right]dx$$

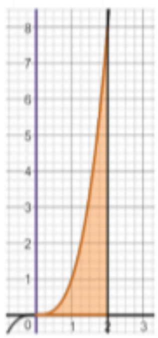
$$\pi\int_{1}^{5}\left[(x)^2-(\sqrt{x})^2\right]dx$$

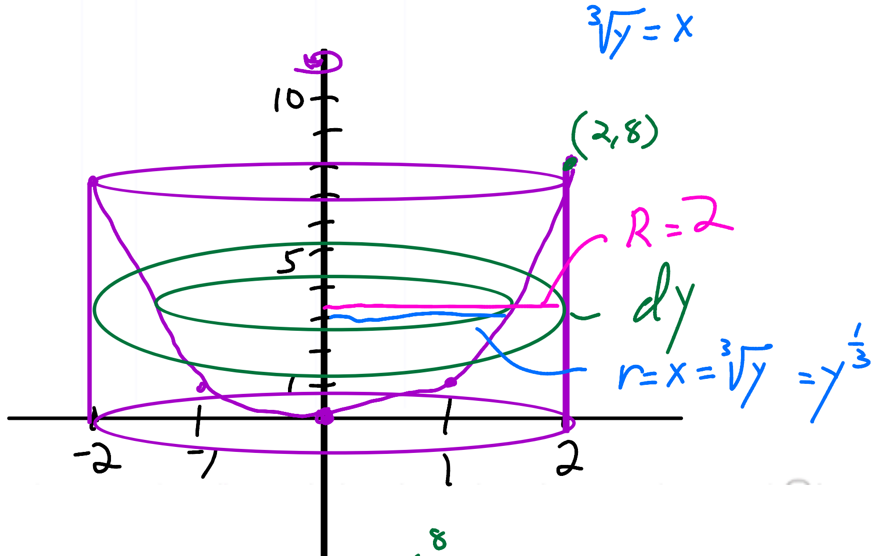
$$\pi\int_{1}^{5}(x^2-x)dx=\pi\left[\tfrac{1}{3}x^3-\tfrac{1}{2}x^2\right]\Big|_{1}^{5}$$

Keep going

8.

Find the volume of the solid generated by revolving the region bounded by $y = x^3$, $y = 0$, and $x = 2$ about the y-axis.





$$\int_0^8 \pi\left[(2)^2 - (x)^2\right] dy$$

$$\pi \int_0^8 \left[4 - (\sqrt[3]{y})^2\right] dy$$

$$\pi \int_0^8 \left[4 - y^{2/3}\right] dy$$

$$\pi\left[4y - \frac{3}{5} y^{\frac{2}{3}+1=\frac{5}{3}}\right]_0^8$$

$$\pi\left[4\cdot 8 - \frac{3}{5}(8)^{5/3}\right] - \cancel{\pi\left[4\cdot 0 - \frac{3}{5}(0)^{5/3}\right]}$$

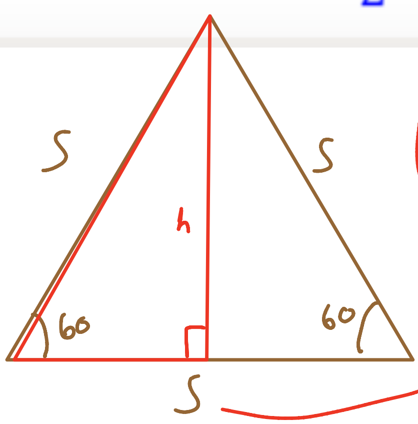
$$\pi\left[32 - \frac{3}{5}\cdot 32\right] = \pi\left[32 - \frac{96}{5}\right] = \pi\left[32 - 19\tfrac{1}{5}\right]$$

Area of a Square = s^2

Area of a Triangle = $\frac{1}{2}bh$

Area of an Equilateral Triangle = $\frac{\sqrt{3}}{4}s^2$

Area of a Semi Circle = $\frac{\pi r^2}{2}$



$Area = \frac{1}{2} b \cdot h$

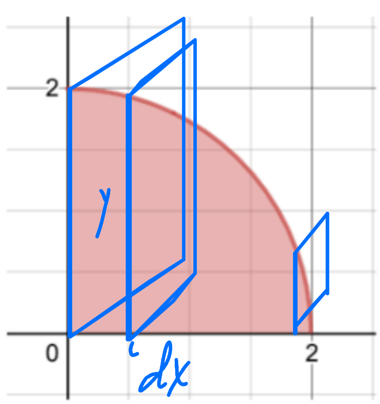
$\sin 60 = \frac{h}{S}$

$S \cdot \sin 60 = h$

$S \cdot \frac{\sqrt{3}}{2} = h$

$Area = \frac{1}{2} \cdot S \cdot \frac{\sqrt{3}}{2} \cdot S = \frac{\sqrt{3}}{4} S^2$

Example 1) Find the volume of the solid whose base is bounded by $x^2 + y^2 = 4$ in the first quadrant, the cross sections perpendicular to the x-axis are squares.



$Area = s^2$

$Side = y$

$x^2 + y^2 = 4$

$y^2 = 4 - x^2$

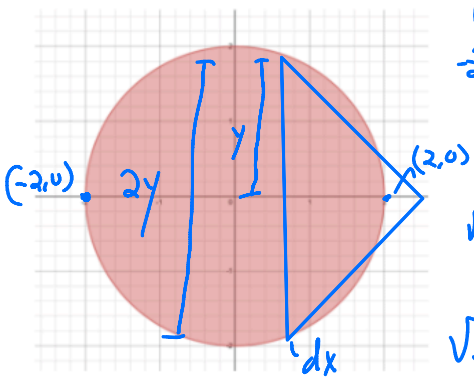
$\int_0^2 s^2 \cdot dx = \int_0^2 y^2 dx = \int_0^2 (4 - x^2) dx$

$4x - \frac{1}{3}x^3 \Big]_0^2$

$= 4(2) - \frac{1}{3}(2)^3 - \cancel{(4(0) - \frac{1}{3}(0)^3)}$

$5\frac{1}{3}$ $\qquad \frac{24-8}{3} = \frac{16}{3} = \frac{3 \cdot 8}{1 \cdot 3} - \frac{8}{3}$

Set up the integral that finds the volume of the solid whose base is bounded by $x^2 + y^2 = 4$, the cross sections perpendicular to the x-axis are Equilateral Triangles.



$$\int_{-2}^{2} \sqrt{3}\, y^2 \cdot dx$$

$$x^2 + y^2 = 4$$

$$y^2 = 4 - x^2$$

$$\sqrt{3}\int_{-2}^{2} (4 - x^2)\, dx$$

$$\sqrt{3}\left[4x - \tfrac{1}{3}x^3\right]\Big|_{-2}^{2}$$

$$A = \frac{\sqrt{3}}{4} s^2$$

$$A = \frac{\sqrt{3}}{4}(2y)^2$$

$$A = \frac{\sqrt{3}}{\cancel{4}} \cdot \cancel{4}\, y^2$$

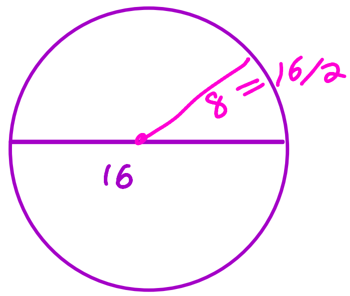
$$A = \sqrt{3}\, y^2$$

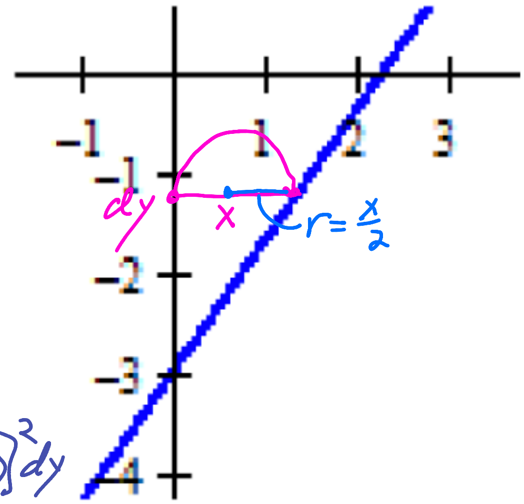
$$\sqrt{3}\left[4 \cdot 2 - \tfrac{1}{3} \cdot 2^3\right] - \sqrt{3}\left[4(-2) - \tfrac{1}{3}(-2)^3\right]$$

$$\sqrt{3}\left[8 - \tfrac{8}{3}\right] - \sqrt{3}\left[-8 + \tfrac{8}{3}\right]$$

$$8\sqrt{3} - \tfrac{8}{3}\sqrt{3} + 8\sqrt{3} - \tfrac{8}{3}\sqrt{3} = \frac{3 \cdot 16}{3 \cdot 1}\sqrt{3} - \frac{16}{3}\sqrt{3} = \frac{(48 - 16)\sqrt{3}}{3} = \frac{32\sqrt{3}}{3}$$

Find the volume of the function $y = \frac{4}{3}x - 3$ bounded by the 4th quadrant if the cross sections perpendicular to the y-axis are semicircles.





$$\int_{-3}^{0} \frac{\pi r^2}{2} dy = \int_{-3}^{0} \frac{\pi x^2}{2} dy = \frac{\pi}{2} \int_{-3}^{0} \left[\frac{3}{8}(y+3)\right]^2 dy$$

$$y = \frac{4}{3}x - 3$$

$$\boxed{\frac{3}{4}(y+3)} = \cancel{\frac{4}{3}}x \cdot \cancel{\frac{3}{4}}$$

$$\frac{3}{4}y + \frac{9}{4} = x$$

$$\frac{\pi}{2} \int_{-3}^{0} \frac{9}{64}(y+3)^2 dy$$

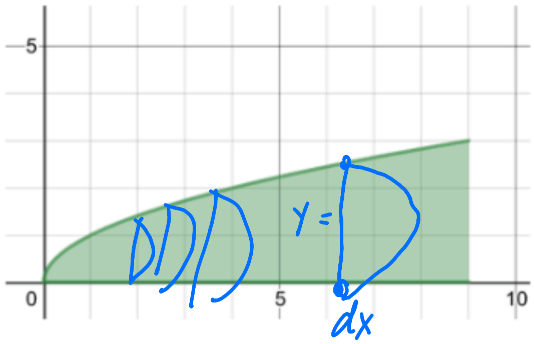
$$\frac{\pi}{2} \cdot \frac{9}{64} \int_{-3}^{0} (y+3)^2 dy$$

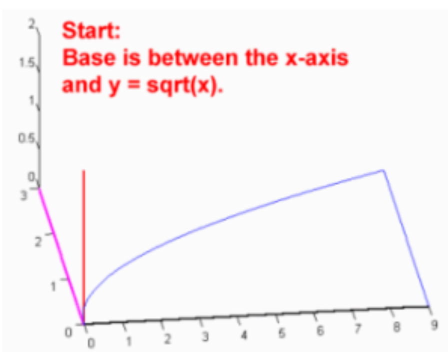
$$\frac{9\pi}{128} \int_{-3}^{0} (y^2 + 6y + 9) dy = \frac{9\pi}{128}\left[\frac{1}{3}y^3 + 3y^2 + 9y\right]\Big|_{-3}^{0}$$

$$\cancel{\frac{9\pi}{128}\left[\frac{1}{3}(0)^3 + 3(0)^2 + 9(0)\right]} - \frac{9\pi}{128}\left[\frac{1}{3}(-3)^3 + 3(-3)^2 + 9(-3)\right]$$

$$-\frac{9\pi}{128}\left[-9 + \cancel{27} - \cancel{27}\right] = \frac{81\pi}{128}$$

Find the volume of the function $y = \sqrt{x}$ bounded by the 1st quadrant and $x = 9$ if the cross sections perpendicular to the x-axis are semicircles.





$r = \frac{y}{2}$

$$\int_0^9 \frac{\pi r^2}{2}\,dx = \int_0^9 \frac{\pi}{2}\left(\frac{y}{2}\right)^2 dx$$

$$\frac{\pi}{8}\int_0^9 y^2\,dx$$

$$\frac{\pi}{8}\int_0^9 (\sqrt{x})^2\,dx = \frac{\pi}{8}\int_0^9 x\,dx$$

$$\frac{\pi}{8}\cdot\frac{1}{2}x^2 \Big|_0^9$$

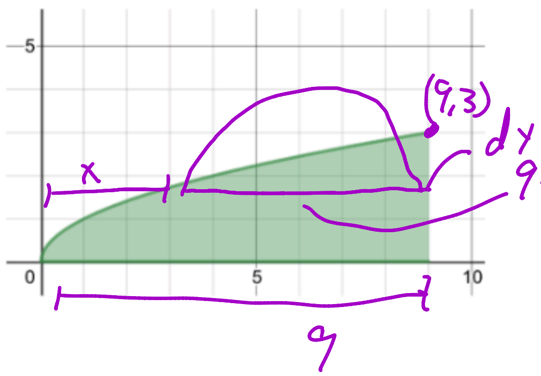
$$\frac{\pi}{16}(9)^2 - \frac{\pi}{16}(0)^2$$

$$\frac{81\pi}{16}$$

Example 3.5)

Set up the integral that Finds the volume of the function $y = \sqrt{x}$ bounded by the 1st quadrant and $x = 9$ if the cross sections perpendicular to the y-axis are semicircles.

$y^2 = x$



$9 - x = \text{dia}$

$r = \frac{9-x}{2}$

$$\int_0^3 \frac{\pi}{2}\left(\frac{9-x}{2}\right)^2 dy$$

$$\frac{\pi}{2}\int_0^3 \left(\frac{9-y^2}{2}\right)^2 dy$$

$$\frac{\pi}{2}\int_0^3 \frac{(9-y^2)^2}{4}\, dy$$

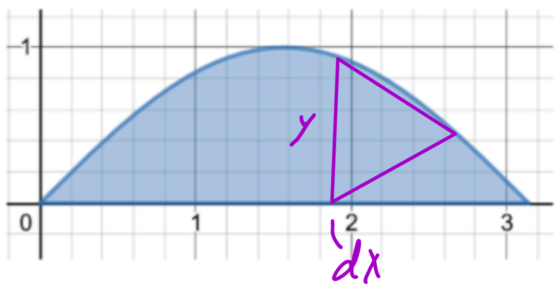
$$\frac{\pi}{8}\int_0^3 (9-y^2)^2\, dy$$

$$\frac{\pi}{8}\int_0^3 (81-18y^2+y^4)\, dy$$

$$\frac{\pi}{8}\left[81y-6y^3+\frac{1}{5}y^5\right]_0^3$$ keep going

Example 4)

Find the volume of the function $y = \underline{\sin x}\ (0 \le x \le \frac{\pi}{2})$ if the cross sections perpendicular to the x-axis are equilateral Triangles.



$$A = \frac{\sqrt{3}}{4}(y^2)$$

$$\int_0^{\pi/2} \frac{\sqrt{3}}{4}(y^2)\,dx$$

$$\int_0^{\pi/2} \frac{\sqrt{3}}{4}\cdot \sin^2 x\,dx$$

$$\frac{\sqrt{3}}{4}\int_0^{\pi/2} \sin^2 x\,dx$$

$$\cos 2x = 1 - 2\sin^2 x$$

$$\cos 2x - 1 = -2\sin^2 x$$

$$\frac{\cos 2x - 1}{-2} = \sin^2 x$$

$$\frac{\sqrt{3}}{4}\int_0^{\pi/2} \left(\frac{\cos 2x - 1}{-2}\right)dx$$

$$-\frac{\sqrt{3}}{8}\int_0^{\pi/2} (\cos 2x - 1)\,dx$$

$$-\frac{\sqrt{3}}{8}\left(\frac{1}{2}\sin 2x - x\right)\Big|_0^{\pi/2}$$

$\sin\pi = 0$ $\quad$ $\sin 0 = 0$

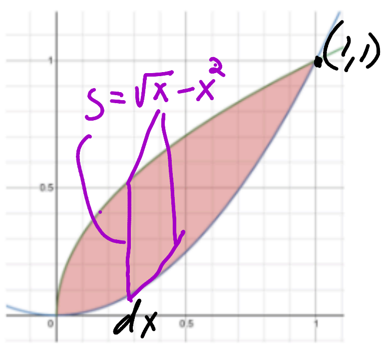
$$-\frac{\sqrt{3}}{8}\left(\frac{1}{2}\cdot \sin 2\cdot\frac{\pi}{2} - \frac{\pi}{2}\right) - \cancel{\frac{\sqrt{3}}{8}\left(\frac{1}{2}\sin 2\cdot 0 - 0\right)}$$

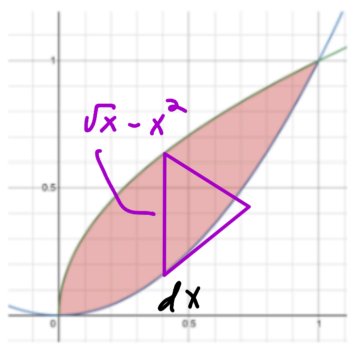
$$\frac{\pi\sqrt{3}}{16}$$

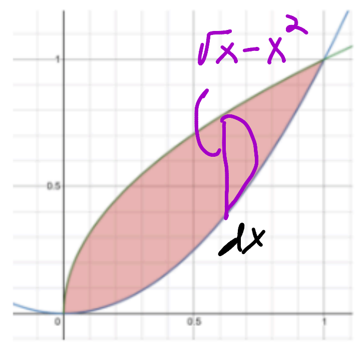
Example 5)

Find the volume of the solid whose base is bounded by $y = \sqrt{x}$ and $y = x^2$, the cross sections perpendicular to the x-axis are a) Squares b) Eq. Tri c) Semi Circles

https://www.geogebra.org/m/XFgMaKTy







$$\int_0^1 (\sqrt{x}-x^2)^2\,dx$$

$$\int_0^1 (x-2x^{5/2}+x^4)\,dx$$

$$\frac{1}{2}x^2 - 2\cdot\frac{2}{7}x^{\frac{5}{2}+1=\frac{7}{2}} + \frac{1}{5}x^5$$

$$\frac{1}{2}x^2 - \frac{4}{7}x^{7/2} + \frac{1}{5}x^5 \Big]_0^1$$

$$\int_0^1 \left(\frac{\sqrt{3}}{4}(\sqrt{x}-x^2)^2\right)dx$$

$$\frac{\sqrt{3}}{4}\left(\frac{1}{2}x^2 - \frac{4}{7}x^{7/2} + \frac{1}{5}x^5\right)\Big]_0^1$$

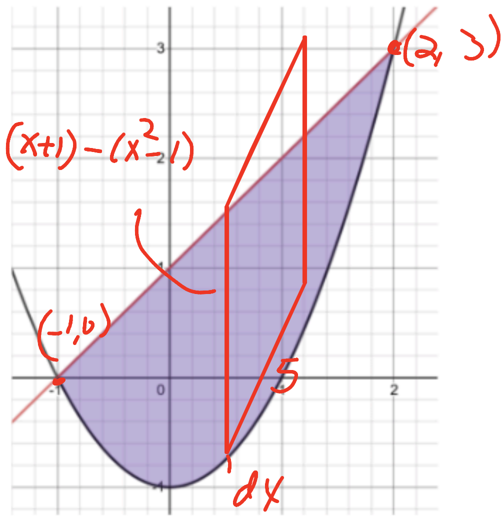
$$\frac{\pi}{2}\int_0^1 \left(\frac{\sqrt{x}-x^2}{2}\right)^2 dx$$

$$\frac{\pi}{2}\cdot\frac{1}{4}\int_0^1 (\sqrt{x}-x^2)^2\,dx$$

$$\frac{\pi}{8}\left(\frac{1}{2}x^2 - \frac{4}{7}x^{7/2} + \frac{1}{5}x^5\right)\Big]_0^1$$

Example 6)

Find the volume of the solid whose base is bounded by $y = x + 1$ and $y = x^2 - 1$, the cross sections perpendicular to the x-axis are rectangles of height 5.



$$\int_{-1}^{2} 5 \cdot \left[(x+1) - (x^2 - 1) \right] dx$$